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$$a = l \sin \phi, \tag{2}$$

$$v^2 = 2g(h - a), \tag{3}$$

$$x = x_1 + a \cot \phi, \tag{4}$$

or,

$$x_1 = x - a \cot \phi. \tag{5}$$

Now taking the top of the inclined plane as the origin, the equation of the path of the body is

$$\begin{aligned} -a &= x_1 \tan \phi - \frac{g}{2}\frac{x_1^2}{v^2 \cos^2 \phi} \\ &= x_1 \tan \phi - \frac{g}{2}\frac{x_1^2}{2g(h-a)\cos^2 \phi} \end{aligned} \tag{6}$$

by equation (3). Whence,

$$(h - a)(a + x_1 \tan \phi) = \frac{x_1^2}{4 \cos^2 \phi}$$

which, putting $x - a \cot \phi$ for x_1 from (5), becomes

$$(h - a)x \tan \phi = \frac{(x - a \cot \phi)^2}{4 \cos^2 \phi}.$$

Hence,

$$x^2 - 2x(a \cot \phi \cos 2\phi + h \sin 2\phi) + a^2 \cot^2 \phi = 0, \tag{7}$$

where x is to be made a maximum.

Differentiating with respect to a, we get

$$\{x - (a \cot \phi \cos 2\phi + h \sin 2\phi)\}\frac{dx}{da} - x \cot \phi \cos 2\phi + a \cot^2 \phi = 0. \tag{8}$$

Hence,

$$\frac{dx}{da} = 0, \quad \text{if} \quad x = a\frac{\cot \phi}{\cos 2\phi}.$$

Putting this value of x in (7) and reducing, we get

$$a = h\frac{\cos 2\phi}{\cos^2 \phi}. \tag{9}$$

Again, substituting this value of x in the coefficient of dx/da in (8), we get

$$a\frac{\cot \phi}{\cos 2\phi} - a \cot \phi \cos 2\phi - h \sin 2\phi,$$

which is nearly equal to $h \sin 2\phi$.

It is evident, therefore, that as a increases through the value $h(\cos 2\phi)/(\cos^2 \phi)$, the coefficient of dx/da remains about equal to $h \sin 2\phi$, a positive quantity, while the remainder of equation (8), namely, $-x \cot \phi \cos 2\phi + a \cot^2 \phi$, changes from $-$ to $+$, for x is constant and a is increasing. Hence, dx/da changes from $+$ to $-$. Hence x is a maximum when

$$x = a\frac{\cot \phi}{\cos 2\phi} = \frac{h}{\sin \phi \cos \phi}. \qquad \text{by (9)}$$

Also, from (2), we get

$$l = \frac{a}{\sin \phi} = 2h\frac{\cot 2\phi}{\cos \phi}. \qquad \text{by (9)}$$

Also solved by O. S. ADAMS and HORACE OLSON.

NUMBER THEORY.

255. Proposed by FRANK IRWIN, University of California.

Given any arithmetical progression whose first term a and common difference d are relatively prime integers, and any finite set of positive integers $m_1, m_2, \cdots$ also relatively prime to d, it is required to determine an integer n such that the multiples of $m_1, m_2, \cdots$ may occupy the same positions in the series of natural numbers beginning with n as they do in the arithmetical progression. This is to say that if the kth, the $(m_1 + k)$th, the $(2m_1 + k)$th, $\cdots$ terms of the progression are divisible by m_1, so also will be the kth, the $(m_1 + k)$th, the $(2m_1 + k)$th $\cdots$ terms of the series $n, n + 1, n + 2, \cdots$, etc. Show that n may be determined as the solution of a congruence $An + B \equiv 0 \pmod{C}$ whose coefficients, A, B, are constants independent of the number and value of the m's.

SOLUTION BY ELIJAH SWIFT, University of Vermont.

By the conditions of the problem the following equations hold: (1) $a + (k - 1)d = m_1\alpha$, (2) $n + k - 1 = m_1\beta$, where α and β are integers. Eliminating k, we obtain,

$$dn - a = m_1(\beta d - \alpha). \tag{3}$$

Consequently we see that n must be a solution of the congruence $dn - a \equiv 0$ (mod L.C.M. of $m_1, m_2, \cdots$). Conversely any solution of this congruence satisfies the given condition. For let n be such a solution. Then (3) is true. We are to show that if (1) holds, (2) will also: and conversely. But adding the equations (1) and (3) we see that $d(n + k - 1)$ is divisible by m_1, and since d is prime to m_1, $n + k - 1$ is divisible by m_1. Similarly (2) implies (1).

Also solved by C. F. GUMMER and HORACE OLSON.

QUESTIONS AND DISCUSSIONS.

SEND ALL COMMUNICATIONS TO U. G. MITCHELL, University of Kansas, Lawrence.

DISCUSSION.

RELATING TO A CONSTRUCTION FOR THE GRAPH OF A CUBIC.

BY PAUL CAPRON, U. S. Naval Academy.

If a cubic is given by the equation

$$\alpha y = x^3 + \beta x^2 + \gamma x + \delta, \qquad (\alpha > 0)$$

and the origin is shifted to (x_0, y_0), where

$$x_0 = -\frac{\beta}{3}, \qquad y_0 = \frac{1}{27\alpha}(2\beta^3 - 9\beta\gamma + 27\delta),$$

the new equation is

$$\alpha y = x^3 + (\gamma - \beta^2/3)x.$$

It is convenient to distinguish three cases, according to the value of $(\gamma - \beta^2/3)$.

Case (1). $\gamma - \beta^2/3 < 0$. Write $\alpha = a^2$, $\gamma - \beta^2/3 = -c^2$; then the equation is

$$a^2y = x^3 - c^2x.$$

$dy/dx = 1/a^2(3x^2 - c^2)$. At the inflection, I, $(0, 0)$, we have $dy/dx = -c^2/a^2 = 1/3\alpha(3\gamma - \beta^2)$, and $dy/dx = 0$ at $[\pm c/\sqrt{3}, \mp (2/3\sqrt{3})(c^3/a^2)]$.

At $(\pm c, 0)$, $dy/dx = 2(c^2/a^2)$, numerically twice the slope at I.

$x = a^2/k$, $y = (a^4/k^3) - (c^2/k)$ are parametric equations for the graph.

Hence the following construction for the graph in Case (1):